RePoSS: Research Publications on Science Studies

RePoSS #8:

Mean Motions in the *Almagest* out of Eclipses

Kristian Peder Moesgaard

May 2010

Centre for Science Studies, University of Aarhus, Denmark
Research group: History and philosophy of science

Please cite this work as:

Kristian Peder Moesgaard (May 2010). *Mean Motions in the* Almagest *out of Eclipses*. RePoSS: Research Publications on Science Studies 8. Aarhus: Centre for Science Studies, University of Aarhus. URL: `http://www.css.au.dk/reposs`.

Mean Motions in the *Almagest* out of Eclipses

by Kristian Peder Moesgaard*

---

*Bygaden 1, Torrild, DK 8300 Odder, Denmark. E-mail: retepeter@mail.tele.dk

1.1: Synodic period relations for the five planets go with eclipse intervals

More than twenty years ago I discovered behind the *Almagest* tables of mean planetary motions a set of synodic period relations linking **P** synodic periods of each planet to **Q** synodic months (KPM: 1985 & 1987)

	Saturn	Jupiter	Mars	Venus	Mercury
P syn.per.	66 + 7/15	199	90	84	329
Q months	851	2688	2377	1661	1291

The discovery included a time measure counting synodic months as its basic time unit, *Ancient Ephemeris Time*, (KPM: 1983 a & b). Note that this does away with any concern about equation of time. Moreover it proved necessary to work with the day to month measure Ms(A) = 29;31,50,20 days, close to the Babylonian system A value, but almost 5 seconds above the system B value Ms(B) = 29;31,50,08,20 days. The latter is otherwise accepted as canonical in the *Almagest.* The difference accumulates to 1,6 hours per century corresponding to a shift of 49' in lunar elongation.

Since 2005 it has been known how the *Almagest* rates of planetary motions were calculated to the fabulous accuracy of a second of arc per 35000 years, far beyond any conceivable empirical foundation (Jones & Duke: 2005). The calculation is about a smaller number of synodic periods, and fits in with the period relations above

	Saturn	Jupiter	Mars	Venus	Mercury
Syn. motion	57*360	65*360	37*360	5*360	145*360
Time, Ms(A)	851*57/66;28	2688*65/199	2377*37/90	1661*5/84	1291*145/329
Time, days	21551;18	25927;37	28857;40,39	2919;40	16802;24
do.			28857;43		
Time, years		71 – 4;53,48 d	79 + 3;11,27 d		
Sun motion		71*360 – 4;50°	79*360 + 3;08,42°		
Planet mot.		2155;10°	15123;08,42°		
do.			15123;10°		

The tables of anomaly result directly from the scheme as the synodic motion per day for Saturn, Venus and Mercury. In the case of Jupiter the scheme produces its rate of motion in longitude, which then subtracted from the solar rate of longitudinal motion leads to the daily anomalistic motion of the planet. The same is true also for Mars, but only after "proportional" augmentation of the time and planetary motion involved. The year and solar rate of motion in question were used by Hipparchus and by Ptolemy in the *Almagest,*

year = 365;14,48 days, w = 360/365;14,48 = 0;59,08,17,13,12,31°/day (1)

and e.g. the anomalistic rate of motion for Mars becomes: W – 15123;10/28857;43.

Much to my surprise the said period relations proved to be eclipse intervals. So they seem the obvious outcome of catching sight of planets close to eclipses.

	Saturn	Jupiter	Mars	Venus	Mercury
P syn.per.	66 + 7/15	199	90	84	329
Q months	851	2688	2377	1661	1291
Drac. Rev.	923.5	2917	2579.5	1802.5	1401
+ d;mm°	0;40	2;30	3;57	3;50	4;17
Ecl. Int.	35	9	6	6	5

E.g. The Saturn period equals 5923/5458*851= 923,5 draconitic revolutions + 0;40°. So the corresponding series of eclipses may continue for 35*851 months, or more than one and a half millennium. The ratio between draconitic and synodic months, 5923/5458 of Babylonian origin was used by Hipparchus and – after a modest adjustment – also by Ptolemy.

1.2: A relation for Mercury out of two solar eclipses

Hereafter any full moon shall be identified by its Goldstine Number, GN, and the next following new moon by the same number + ,5 (Goldstine:1973). In this calendar Nabonassars epoch equals GN 3142,69616, where the fraction reflects the *Almagest* radix of lunar elongation, 250;37/360 = 0,69616.

Until recently I considered indentifying specific eclipses behind the period relations an impossible job, say like searching five pairs of pins in a field with haystacks in scores most of which had possibly disappeared ages ago. But then I stumbled on the Mercury interval of 1291 months between two reports of solar eclipses,
-240NO28, GN = 9410,5 An. = x*360° + y° (Sachs & Hunger 1989, p. 79)
-135AP15, GN = 10701,5 An. = (x + 329)*360° + y° (KPM 1987, p. 52)
Extrapolating to Nabonassars Epoch yields An. = (x - 1598)*360° + y° + 253;54,23° very close to the same number of minutes of arc as in the *Almagest* radix, 21;55°. As a matter of fact, with x = 1597 and y = 128° we get An(Nab.) = 21;54,23°. And the most compendious expression for the anomaly of Mercury runs:

$$\mathbf{An.(GN) = 128^o + (GN - 10701{,}5)*360*329/1291} \qquad \mathbf{(2)}$$

1.3: For the remaining planets eclipses emerge from similar relations

Guided by the Mercury result I searched out relations for the other planets combining their *Almagest* radices with integral degree values of the anomaly for new- and/or full moons. Any hit creates a series of possible starting points, e.g. in the case of Mercury above, GN = 10701,5 +/- any number of the period 1291 months. Now the inner planets are always seen close to the Sun whence they can never witness a lunar eclipse in opposition to the Sun, and looking for full moon solutions would be idle. Not all hits prove useful. E.g. the relation An(GN) = (GN – 9563)*360*90/2377 for Mars produces the right radix, 327;14° (327;13° in *Almagest*). But here the zero anomaly, i.e. Mars in conjunction with the Sun, is linked with full moon and possibly a lunar eclipse, but of course without the presence of Mars. Most promising are the following relations:

An.(GN) = 89º + (GN – 10677,5)*360*84/1661 **(3, Ven.)**

An.(GN) = 355º + (GN – 10196,5)*360*90/2377 **(3, Mrs.)**

An.(GN) = 359º + (GN – 9985,5)*360*199/2688 **(3, Jup.)**
An.(GN) = 165º + (GN – 10478)*360*199/2688 **do.**

An.(GN) = 160º + (GN – 9997)*23928º/851 **(3, Sat.)**

The case of Jupiter involves 2688 steps per 360º, or 112 per 15º. So any hit creates a set of 24 members 15º apart. On the condition, however, that Jupiter be visible close to an eclipse we are led to the two solutions above. The Saturn period of 851 months goes with a change in anomaly of 7/15*360º = 168º. This will move the planet from conjunction to the neighbourhood of opposition or vice versa. So the useful period between eclipses witnessed by Saturn becomes 2*851 months with a backwards shift in anomaly of 24º.

The actual eclipses to go with equations (2) & (3) from the last half millennium BC, are collected in the following scheme. The top items for Mercury, Venus, Mars and Jupiter (at full moon) initiate eclipse series which continue for centuries after the Birth of Christ. The Jupiter column (at new moon) almost ends a series that began more than one and a half millennium BC. For Saturn GN 9997 occurs right in the middle of a series of 35 eclipses from 1363 BC to AD 977.

Below the names of the planets with the additon of ”n” for new and ”f” for full moons you find two lines of radices corresponding to Nabonassars Epoch, first (in grey shading) as calculated from my tables of reference (KPM: 1975c) and then the tabular entries in the *Almagest*. The equations (2) & (3) above ensure the same results within the clearance of one arcminute. The parameters of the said equations end the heading, including three radices for Saturn to go with each of the three entries below.

The eclipses are presented by their GN-number, the local hour of the day in Babylon (Goldstine: 1973), and the year for easy reference to the astronomical diaries (Sachs and Hunger: vols. 1-3). The local hour is relevant for making out the visibility of an eclipse. E.g. the solar eclipse GN = 9985,5 occurred about an hour after midnight and could certainly not be observed in Babylon. Yet it is mentioned in the diaries as shown in the scheme by the background colour: ”Night of the 29^{th}, at 1,17º after sunset, solar eclipse which was omitted.” Even the information that somebody was on the lookout for the eclipse may prove important.

Mercury	Venus	Mars, n	Jupiter, n	Jupiter, f	Saturn, f
11;26	68;02	327;12	144;44		33;12
21;55	71;07	327;13	146;04		34;02
329/1291	84/1661	90/2377	199/2688		(66 8/15)/851
128	**89**	**355**	**359**	**165**	**208//184//160**
		5442,5			**6593**
		-560//07;33			-467//06;14
		30k + 9,1	**7297,5**	**7790**	13k + 2,6
			-410//12;00	-371//12;18	
		7819,5	29k – 4,8	5k + 10,0	**8295**
		-368//07;48			-330//21;27
	9016,5	1k + 5,2			34k + 1,3
	-271//04;54				
9410,5	28k + 12,5		**9985,5**		**9997**
-240//14;46			-193//00;49		-192//01;58
23k – 11,7		**10196,5**	5k – 7,3	**10478**	20k + 0,0
		-176//17;55		-153//20;12	
10701,5	**10677,5**	7k + 1,2		16k + 7,5	
-135//09;10	-137//17;45				
1k – 7,4	3k + 8,7				

The third line of an entry tells the approximate location (loc.) of the eclipse. Thus ”13k + 2,6” below GN 6593 means that an eclipse of the moon occurred near knot nr. 13 of the *full moon serpent* and 2,6º from a node of the lunar orbit. So the latitude of the moon was about a quarter of a degree, and its longitude close to the middle of the Lion, (KPM:1980, p. 52, Figure 1). Solar eclipses are located similarly in a *new moon serpent*. Eclipse warnings result with the distance to a lunar node kept below about 11,8º. The numbers are most compendiously determined by counting the knot distancies passed by the moon since it hit dead on knot 20 at GN 9997 (KPM:1980, cf. eqn. 25, p. 69 and Figure 3, p. 64).

loc.(GN) = 20 + (38 – 465/2729)*(GN – 9997) modulo 35 (4)

So loc.(6593) = -128751.9853 = -3679*35 + 13k + 0,0147*180º. The equation (4) being linear it cannot consider the effect of solar anomaly, whence away from knot 20 the outcome is average and approximate. The factor chosen here reflects the Babylonian relation: 5458 months = 5923 draconitic revolutions.

2: Suggestions for the move from eclipses to the synodic relations

Possibly some of the eclipses above produced the relations (2) & (3) as indeed we saw for Mercury. Yet the empirical basis may have included a wider choice of eclipses and arguments.

As an example take *the argument of symmetry*: The diaries mention solar eclipses in the years -248 and -280 (Sachs, Hunger: 1988 p. 313 & 1989 p. 51).

Now the equation (2) produces the symmetrical anomalies: An.(8905,5) = 237;52 = -122;08 and An.(9304,5) = 123;16. Shifting the radix to 127 leads to -123;08 and 122;16; whereas other choices destroy the symmetry. So the possible observation of symmetric situations and the demand for integral degree radices leaves you with the choice between 127 and 128.

Another piece of decisive information may result from *close encounter* observations: E.g. The Mercury column eclipse, GN = 10701,5, revealed “invisible” Jupiter close to the Sun during its darkness (KPM: 1987, p. 52). This occurred 3404, or 716 + 2688, months after the Jupiter column eclipse, GN = 7297,5. So the corresponding net change of anomaly was 2;41º. Say Jupiter was seen to the opposite side, but again close to the Sun. Then you needs must put its anomaly to -1º (= 359º), or perhaps -2º, and you got a good reason for being on a midnight(!) lookout for the next eclipse in the Jupiter column, GN 9985,5.

Thirdly arguments may flow from *periods of multiple hits*. Thus the Saturn period of 851 months also works for Jupiter, 199*851/2688 = 63 + 0;40º/360. Note also that a period of 878 months comes close for Jupiter, 199*878/2688 = 65 + 0;16º/360, and in the bargain hits the *goal year period* of 71 years, 878*19 = 71*235 – 3. The latter could perhaps bear on the long-winded calculation of the daily rate of motion for Jupiter (cf. p. 1 above). The synodic period of Jupiter is only a few hours longer than 13,5 months whence 10 periods are close to the eclipse interval of 135 months: 199*135/2688 = 10 – 2;00,32º. But this means that our prototables produces further integral degree anomalies. And indeed the lunar eclipses GN 6311 and GN 7655 go with the anomalies of Jupiter 347;01º and 167;01º. They are used in the *Almagest* (nos. 7 & 9 in Pedersen: 1974, p. 409), they are 2688/2 months apart, and the latter occurs 135 months earlier than GN 7790 in our sheme above.

The Mars period 581 months, 90*581/2377 = 22 – 0;36º/360, restores both Jupiter's anomaly, 199*581/2688 = 43 + 4;41º/360, and the longitude, 581*19 = 47*235 – 6. The Mercury period 569 months, 329*569/1291 = 145 + 1;40º/360, also comes close to an integral number of years, 569*19 = 46*235 + 1.

3: Evaluating the genesis of the *Almagest* planetary tables

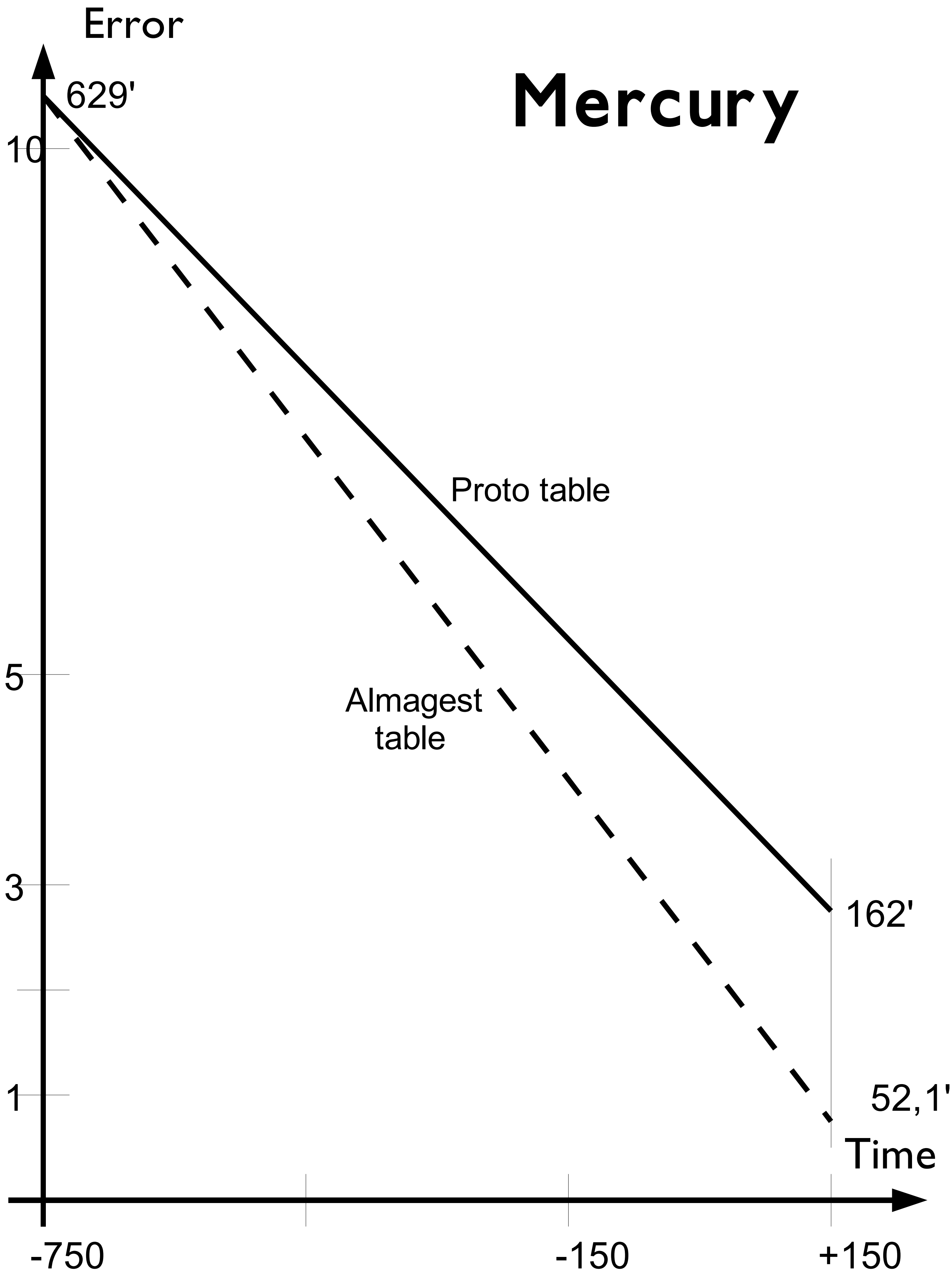
Even if we have not so far spotted the complete observational basis behind the synodic relations it is very likely that exactly these relations represent the natural outcome of keeping track of the planets near eclipses for at least a couple of centuries. Now we want to evaluate the relations themselves, and their offspring viz. the *Almagest* planetary tables. The next scheme puts this into effect.

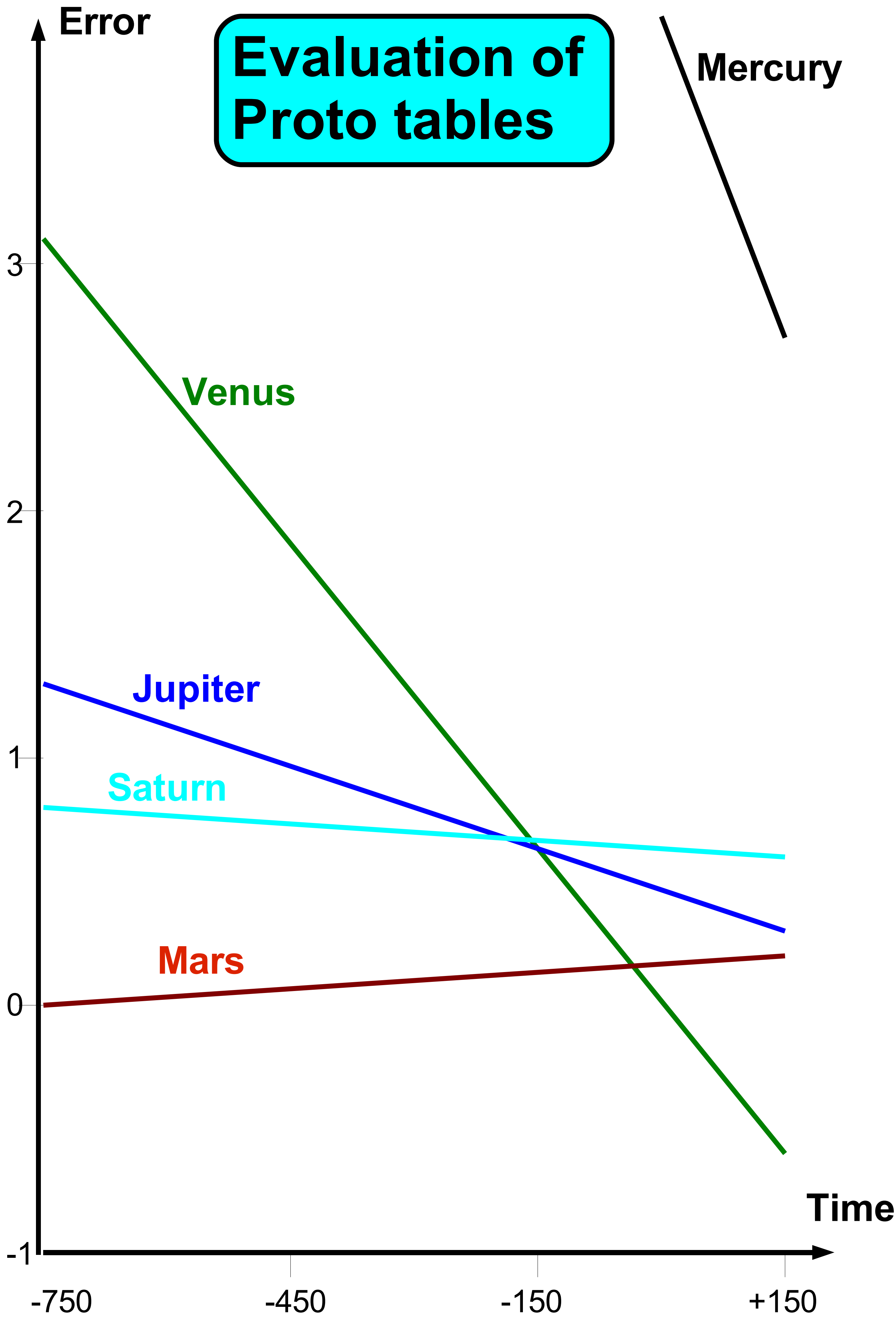
The midmost column evaluate the common radices at Nabonassars epoch for the equations (2) & (3) on one hand and for the *Almagest* tables on the other. Reference values are drawn from my tables (KPM:1975c, & cf. The scheme on p. 3 above).

	Equations (2) & (3)		Almagest tables
	Rate º/Ms	**Radix, Nab.**	**Rate º/day**
Saturn	23928/851	34;02º	0;57,07,43,41,43,40
Error	-1,4'/cent.	+0;50º	-5,1'/cent.
Jupiter	71640/2688	146;04º	0;54,09,02,46,26,00
Error	-7,0'/cent.	+1;20º	-10,3'/cent.
Mars	32400/2377	327;13º	0;27,41,40,19,20,58
Error	+1,4'/cent.	+0;01º	-0,5'/cent.
Venus	30240/1661	71;07º	0;36,59,25,53,11,28
Error	-24,8'/cent.	+3;05º	-27,1'/cent.
Mercury	118440/1291	21;55º	3;06,24,06,59,35,50
Error	-51,9'/cent.	+10;29º	-64,1'/cent.

The large error in the case of Mercury is natural since observations of the inner planets must take place in some distance from the Sun where their motion is to some extent radial (as opposed to angular and directly observable).

The columns to the left and to the right evaluate the rates of motion as determined from our equations (2) & (3) and as found in the *Almagest* respectively. The error values are averaged over the period from 300 BC to AD 100 as compared to my tables of reference (KPM:1975c). E.g. The four centuries go with 4947,41134 Ms and 187,32285 synodic periods of Mars. So the deviation is 4947,41134*90/2377 – 187,32285 = 5,51', or 1,4'/century. And for the right hand error we get: Daily rate*4*36525 minus 187,32285*360 equal to -2,1', or – 0,5' per century. The difference in reliability between the two columns hinges on the shift of time measure from months to days, and in particular on the use of the system A month, Ms(A), longer than Ms(B) by the accumulated time of 1,6 hour per century. E.g. The Saturn motion during 1,6 hour makes 3,8' to go with the difference between -1,4' and -5,1' in the first error line. By hindsight we know that Ms(B) was correct to within a tenth of a second during some centuries around the Birth of Christ. You cannot assume the ancient astronomers to have been able to decide between the two month measures. But you can accuse them for mixing up both in the same textbook.


Error
Mercury
629'
10
Proto table
5
Almagest
table
3
162'
1
52,1'
Time
-750
-150
+150


Error
Evaluation of Proto tables
Mercury
3
Venus
2
Jupiter
1
Saturn
Mars
0
Time
-1
-750
-450
-150
+150

The graphic representations of the tabular errors evidently show Mercury as a separate case. Here we possibly face the full story originating in the difficulty of observing the angular displacement of that planet, because it is always found in the neighbourhood of the Sun. Thus the supposed symmetric equation (2) anomalies

An.(8905,5) = 237;52 = -122;08 and An.(9304,5) = 123;16

are too high by about 6 degrees. So you cannot hold by the choice of symmetry. And to match the standard of equations (3) for the other planets, the radix of equation (2) should be lowered to 122, or 123. The gap between the tables at AD 150 reflects the shift from Ms(A) to Ms(B) for the month to day measure.

The prototables for the remaining 4 planets proves a world more reliable than the Mercury table. As suggested above they also rest on a much more solid foundation of eclipse observations. It appears that they were possibly put together sometime during the second century BC. And naturally Hipparchus comes to mind as the probable originator of these tables. But with the authorship unknown I content myself mentioning “Hipparchus, or another contemporary, but so far Unidentified Hellenistic Astronomer (UHA)”. Anyway with such tables at hand generations before Ptolemy the task of refining upon the planetary theories in the *Almagest* was facilitated a lot.

4: Mean motions of Moon and Sun

Here prototables include the choice of basic time unit, the synodic month. The *Almagest* uses Ms(B) quoted already at the outset (cf.p.1). Its day measure originates in Babylonian lunar theory, Hellenistic astronomers held it canonical, and through the Renaisance it was still in use. By hindsight we judge the measure correct to within a tenth of a second between 200 BC and Ad 500. I wonder how the ancient astronomers determined Ms(B), and whether they could decide how good it was. Anyway the corresponding daily increase in lunar elongation is a basic parameter in the *Almagest:*

360º/29;31,50,08,20 døgn = 12;11,26,41,20,17,59 º/day = ω (5)

Another basic parameter is the year of 365;14,48 days and its offspring, the solar rate of motion, w of equation (1). But even more fundamental is the ratio ω/w with the continued fraction convergents: 12, 25/2, 37/3,99/8, 136/11, **235/19**, 123511/9986. So hitting closer than 235/19 demands a time span of almost 10000 years. Thence I take the relation **“19 years = 235 months”** to define a “synodic” year and its “synodic” longitude measure. This ensures repetition of 235 possible positions with average steps of 360º/235 between them, and 19 steps as the monthly motion. The period is well-known from ancient Babylon through our Christian Easter calculation.

	Rev./month	Error '/cent.	Table, º/day	Error '/cent.	Final error in *Almagest, '/cent.*	
Moon						
Elongation	1	Unit	(cf. equ. 5) ω	0,7'		
Draconitic	5923/5458	1,6'	5923/5458*ω	2,3'	3,8'	
Anomaly	269/251	11,4'	269/251*ω	12,2'	10,2'	
Sun						
Synodic	19/235	Definition	(cf. equ. 1) w	1,0'	Tr: -24,7'	An: 77,4'
Sidereal	160/1979	-0,1'	160/1979*ω	-0,0'		
Anomaly	225/2783	-5,3'				

I have added the sidereal year, 1979/160 = 12;22,07,30 months. Combined with the synodic year, correct by definition, it produces **precession close to a degree per century,** certainly a canonical rate in antiquity. So synodic longitude together with 1º per century "precession" produces a sound sidereal doctrine of astronomy.

Combining equation (1) with the equations (2), (3) and (5), creates a complete set of motions in longitude: for the Sun and the inner planets w will do, ω plus w for the Moon, and w minus the anomaly for the outer planets. The results are of course "synodic". So strictly we can only fix full moon (and solar) positions in relation to – exactly! - other full moons. The 235 full moon locations constitute a slowly turning dial. But knowing its rate of drift leads to a set of reliable sidereal longitudes – apart from the case of Mercury where the sloppy anomaly shall already destroy the result.

The right hand error columns of the scheme above show that the parameters remain sound after changing the time measure from months to days, and even after Ptolemy's adjusting the lunar draconitic motion by +9' during 615 years between observations no. 7 and no. 54 and the lunar anomaly by -17' during 853 years between observations no. 2 and no. 69 (Pedersen:1974, p. 180; numbers from list, p. 408 ff.). Finally the real **scandal of Hellenistic astronomy** stares us in the face. Equating synodic longitudes with **tropical** ones produces an error of almost half a degree per century, and even worse in the case of **anomaly** where the error is well above a degree per century. We shall soon reveal how all this came about.

In the sequel we shall ignore lunar anomaly. It is certainly instrumental for timing eclipses. But it has only minor influence on their locations. Full moons and lunar eclipses occur ***when*** the Moon arrives at the right position which may be determined independently, by and large with reference to annual anomaly only. To complete the list we mention the year of 2783/225 = 12;22,08 months. This is Babylonian system A year, and together with the draconitic month = 5458/5923 synodic months, it forms the backbone of column E, a really fully fledged eclipse theory,which evidently has not influenced the doctrine of the *Almagest.*

5: The crime(?) of Hellenistic astronomy

As foundation stone of the *Almagest* you find solar motion at the rate 360º/365;14,48 days in ***an eccentric circle.*** This is meant to reflect both ***tropical***, or seasonal, and ***anomalistic***, or orbital motion of the Sun. Equating the two rates means assuming ***an apsidal line of symmetry, fixed in relation to the cardinal points, solstices and equinoxes***. And we are told how the rate of motion results from observations of solstices and equinoxes as related in the scheme below.

Selected *Almagest* solstices and equinoxes

No.	Year/date/hour	Julian Day(Ut)	Error(h)	Astronomer	Aut.	Spr.	Sum.
8	-431JN27 06	1563813/04	-27,6	Meton et al.			0
16	-279(JN26 18)	(1619330/16)	-10,3	Aristarchus			152
37	-146SE27 00	1668000/22	7,2	Hipparchus	0		
38	-145MR24 06	1668179/04	-8,2			0	
44	-134/JN26 12)	(1672291/10)	5,7				297
89	139SE26 07	1772096/05	34,4	Ptolemy	285		
91	140MR22 13	1772274/11	21,5			285	
92	140JN25 02	1772369/00	37				571

The left hand numbers refer to the list in "A Survey of the *Almagest*" (Pedersen:1974, p. 408 ff.). Between the solstices by Meton and Aristarchos we count 152 = 8*19 years and 55517,5 days, i.e. 1 year = 365;14,48,09... days naturally rounded to the tabular 365;14,48. The difference goes with a change in solar motion of 15" per century. The very same year is also ascribed to Sudines, and together with Aristarchos' sidereal year of 55519/(8*19) days it leads to the "precession" 58' per century (KPM:1983b, p. 57). **Solar tables with daily rate of 360º/365;14,48 coupled to 1º/century precession may have been at hand already shortly after 300 BC**. Still another synodic year is ascribed to Aristarchos, 365;14,48,54... = 29;31,50,10*235/19, combining the 19 year period and a third value of the month which I have not found attested elsewhere.

If not before, then at the latest in the second century BC the synodic longitudes were finetuned with reference to two eclipses mentioned in the *Almagest*. Between GN6182 (-501NO19) and GN7667 (-381DE12) we count 1485 months, and therefore 1485*19

= 120*235 + 15 steps, or 120*360º + 23º (to within a minute of arc). So the adjusted year becomes Y = 1485*Ms(B)/(120 + 23/360) = 365;14,47,54... days naturally rounded to the tabular 365;14,48. The difference goes with a change in solar motion of 9" per century. But furthermore the finetuned year creates the key intervals between solstices and equinoxes put forward in the *Almagest* for building a tropical(!) solar theory, say the time intervals from Meton to Hipparchus and to Ptolemy:

t(44) – t(8) = 108478,25 days = 297*Y – 4 min. (or -0.8 sec./year).

t(92) – t(8) = 208555+5/6 days = 571*Y – 45 sec. (or -0.08 sec./year).

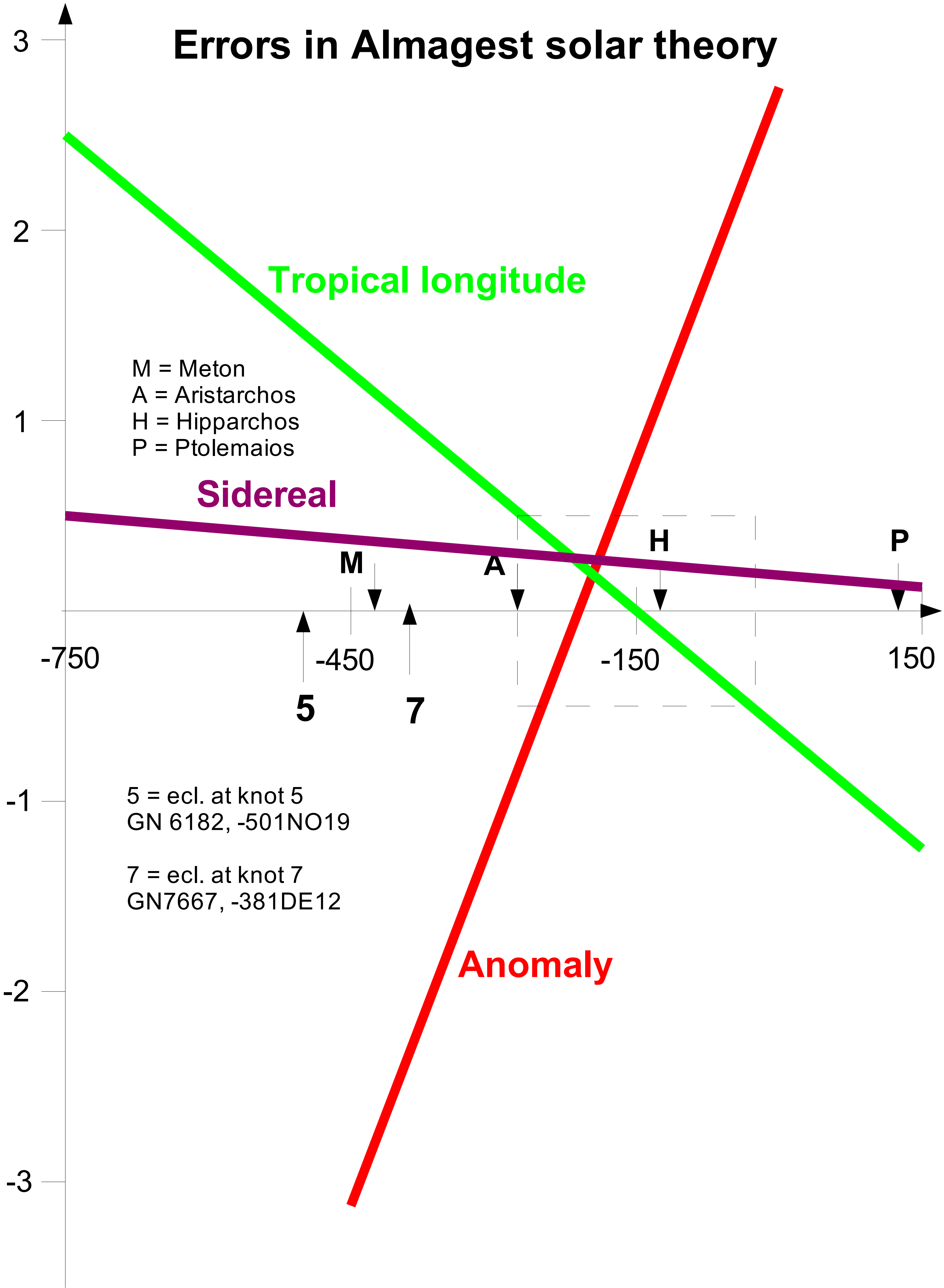
Also the time spans between equinoxes by Hipparchus and by Ptolemy result:

t(89) – t(37) = t(91) – t(38) = 285*Y – 2 min. (or – 0,4 sec./year)

Perhaps the Meton solstice was reported as quoted, too early by more than 24 hours. In this case a tropical year of the order of magnitude 365;14,48 days was given to Hipparchus and he may have acted in good faith equating the synodic and the tropical year. Otherwise some kind of fabrication must have been involved. Anyway the tabular rate of equation (1), perhaps already in use since Aristarchos, received strong additional support from the said eclipse data.

But also the 1º/century rate of "precession" gains credibility from lunar eclipse data. The 235 possible full moon positions include 35 possible domiciles for eclipses. Generally, in each of these, a lunar eclipse triad occurs, say at month no. 0, 235 and 470 whereas you will look out in vain for a fourth eclipse at month no. 705. But then at month no. 804 we face the initial member of a new eclipse triad at the neighbouring position, 804*19 = 65*235 +1, and 804*5923/5458 = 872,5 rev. – 0;51,27º. So all around the Zodiac eclipses occur in triples drifting at the rate 1 step = 360º/235 every 65 years, or 2;21º per century. But observing the drift of eclipses among the stars reveal a much smaller rate as seen in Figure 3 of my Full Moon Serpent (KPM:1980, p. 64) and shown in the following scheme concerning 4 lunar eclipses mentioned in the *Almagest* and belonging to the same series of triads. The increases in sidereal longitudes are read off the Figure to the nearmost half degree. So inevitably the stars have moved at the rate **a degree per century** in relation to the eclipse pattern dial.

No.	Date	GN	Triad	Years	Longitude Synodic	Increase Sidereal	Diff./cent.
4	-620AP22	4703	6	0	0	0	
7	-490AP25	6311	8	130	3,1°	2°	51'
33	-173AP30	10232	12	390	9,2°	5°	65'
63	133MY06	14017	17	715	16,9°	9,5°	62'



Errors in Almagest solar theory
3
2
1
-1
-2
-3
-750
-450
-150
150
Tropical longitude
Sidereal
Anomaly
M = Meton
A = Aristarchos
H = Hipparchos
P = Ptolemaios
M
A
H
P
5
7
5 = ecl. at knot 5
GN 6182, -501NO19
7 = ecl. at knot 7
GN7667, -381DE12

Introducing the *Almagest* radices of solar longitude and anomaly we get the tables:

Tropical longitude = 330;45º + w*(t – t(Nab.)) modulo 360º

Anomaly = 265;15º + w*(t – t(Nab.)) modulo 360º

Their deviations from my tables of reference (KPM:1975c) are found in the foregoing graph, and it appears that in Hipparchos' time reasonably reliable results were established both for the seasonal and for the orbital motion of the Sun. However, when Ptolemy three centuries later copied the whole story without changing one jot his *Almagest* came to keep a synodic soul tied to a tropical body only temporarily around the middle of the second century BC (KPM:1983b, p. 59). Furthermore he defends the tropical solution repeatedly:

> *... the only reference point we must consider when examining the length of the solar year is the return of the Sun to itself, that is (the period in which it traverses) the circle of the ecliptic defined by its own motion. ... proper starting-points for the sun's revolution are those defined by the equinoxes and solstices on that circle. ... from a mathematical viewpoint ... returns the sun to the same relative position, both in place and time ... from a physical point of view ... returns the sun to a similar atmospheric condition and the same season ... it seems unnatural to define the sun's revolution by its return to (one of) the fixed stars, especially since the sphere of the fixed stars is observed to have a regular motion of its own ... it would be equally appropriate to say that the length of the solar year is the time it takes the sun to go from one conjunction with Saturn, let us say, (or any other of the planets) to the next. In this way many different "years" could be generated* (Toomer:1984, p. 132).

This raises a swarm of questions. Does it reflect an echo of the foregoing synodic basis for the planetary mean motions? How could the year of 365;14,48 days gain authority enough to overrule blatant errors well above a whole day in timing solstices and equinoxes (see p. 2)? Could Ptolemy really be ignorant of the highly reliable sidereal motions built into the astronomical tradition possibly dating back to at least Aristarchos.

Anyway our graph also evaluates the motion of the sun in relation to the stars. No natural zero-star is at hand, so the error line may be moved up or down. I have chosen with Copernicus as "zero" *a point 170º west of the AD1520 position of Spica.* Thus he made Spica represent the sphere of the fixed stars and so he built **correspondence** in his *De revolutiobus* with the sidereal reality behind the confused tropical data in the *Almagest* (KPM:1974b, p. 260-65, especially Fig. 17 p. 264).

Let us return to Hipparchus' achievement: he was renowned for his investigation of the tropical position of the Sun. And by hindsight we may praise his success. In the graph **the errors of seasonal solar longitude change sign in his time**. This means that after all he chose for his solar tables the right value of their radix, **330;45º**.

In addition he seems to have succeded in establishing the solar anomaly. But surprisingly the anomalistic motion was probably determined independently, viz. from data referring to lunar eclipses. The two eclipses mentioned above are located before and after *the perigee* of the pattern of full moon positions and thence – hopefully – *the apogee* of the solar orbit. We shall corroborate this in the following section. But here **assuming symmetry** we get the anomalies:

An(6182) = 168,5º, An(7667) = 120*360º + 191,5º,

and at Nabonassars Epoch: An(3142,69616) = 265;19º - 246*360º

only 4' above **265;15º** found in the *Almagest*, and probably rounded from this very calculation.

It is tempting to find the **unchangeable(!) longitude of apogee in the solar(!) orbit** as the difference between the two radices, 330;45º – 265;15º = **65;30º**. And probably that was exactly how Hipparchus, and following him Ptolemy, proceeded, even if both of them refer to another foundation, namely durations of annual seasons. If so, it appears that the process of manipulating intervals between equinoxes and solstices to yield a desired result was indeed initiated by Hipparchus.

<u>6: From mean to true motion of the Sun</u>

235 consecutive full moon positions form a nice pattern of 35 sinusoidal waves that span the zodiac twice and by its 35 intersections determine possible eclipse locations. This is the full moon serpent, cf. Figure xx (KPM:1980, p 52 & KPM:1983b, p. 48) which is directly observable, since the full moon "events" time and again reveal themselves among the stars in the shape of eclipses. In the Figure the full moons are numbered, n = 1, 2, 3, ... 235, with no. 1 at the beginning of Aries, and the eclipse locations are numbered similarly, N = 0, 1, 2, ... 34.

From the Figure it is evident that eclipsing "knot" no. 15 is directly opposite to "knot" no. 33, i.e. from Virgo to Pisces you find 18 knot intervals each of 10º on the average, but from Pisces to Virgo only 17 intervals of about 10;35º. Perpendicular to this you meet the axis of symmetry from "knot" no. 24 to somewhere between "knots" nos. 6 and 7, i.e. by Gemini and Sagittarius the zodiac and the full moon serpent are split into halves by the same division. All this is illustrated in the following page where the 235 full moon positions are also divided between 114 steps of 180º/114 = **30º/19** = 1;35º and 121 steps of **28;20º/19** = 1;21º in the two halves of the zodiac respectively. Note that we came to create some key parameters of the Babylonian System A' solar theory.

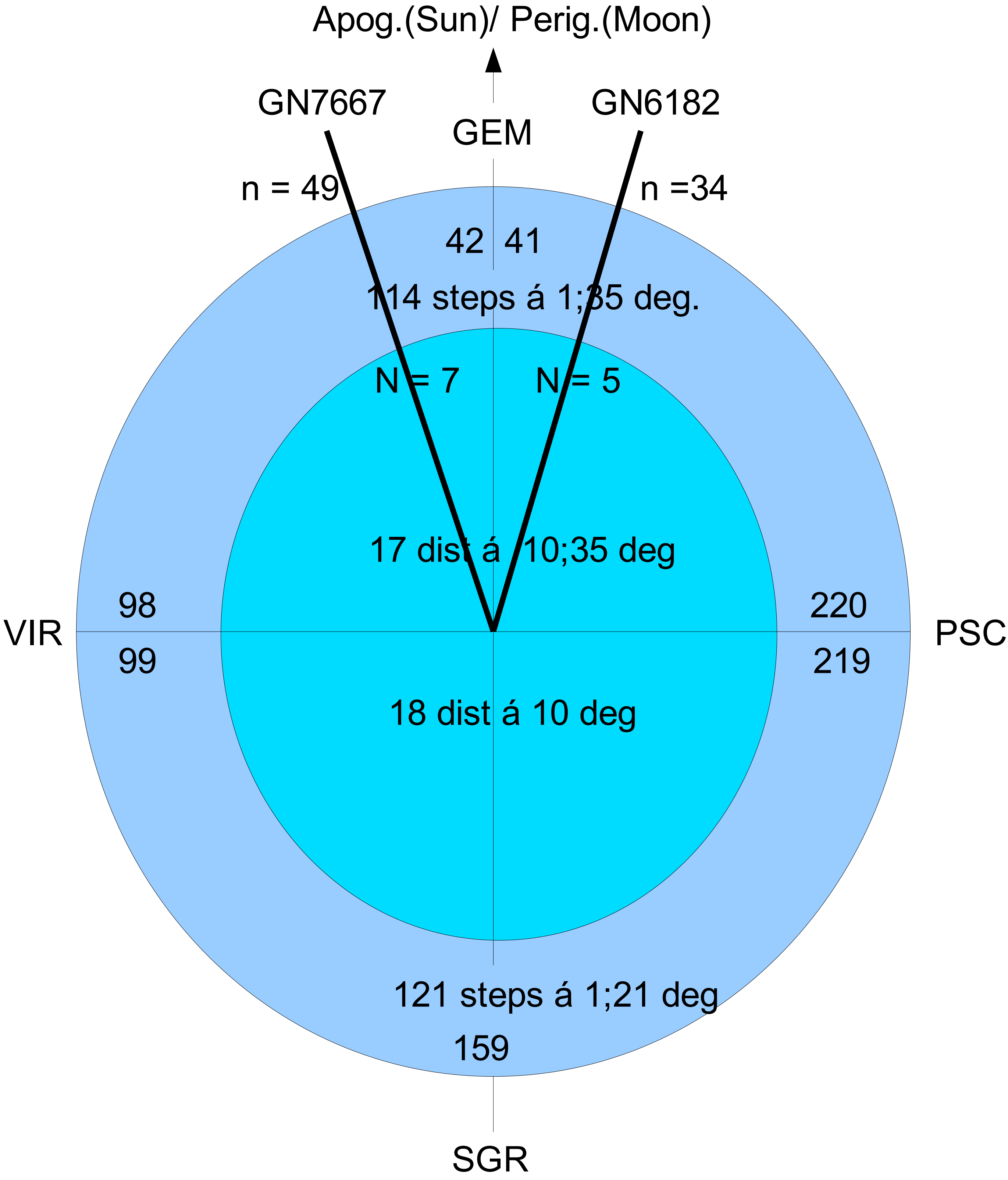


Figure: m-steps and eclipse distances in their skew distribution around the Zodiac

For the entire story se my “Full Moon Serpent” (KPM: 1980, p. 87-92). In our Figure above we have turned the line of symmetry through half a “knot”distance to connect “knot “ no. 6 with a point midway between “knots” nos. 23 and 24. Thus we illustrate the Hipparchian(?) determination of the anomaly and his solution for the longitude of the apogee midway between the n-steps nos. 41 and 42, to witt 41,5*180º/114 = 65,5º.

Assuming **round half degree inequalities** for the two eclipses at “knots” nos. 5 and 7, the true anomalies become 168º and 192º respectively, and the maximum prosthaphairesis 0,5º/sin12º = **2;24º**, very close to the *Almagest* solution, but far too large compared to the actual solar equation of 2º. Other suggestions from the Full Moon Serpent would be one fourth of a “knot” interval 360º/(4*35) = 2;34º , or perhaps one and a half serpent step, 1,5*360º/235 = 2;18º. So after all we would expect an eccentricity stemming from eclipse data to be exaggerated, as it should be because it includes solar anomaly of 2º, annual equation of the Moon , ca. 11', and - at least in one half of the zodiac - a similar contribution from the lack of any equation of time.

Finally the obliquity of a ***sidereal ecliptic*** defined as the path among the stars where central lunar eclipses occur as seen from Babylon, or the circle of apparent zero latitude, is bound to appear more oblique than the true solar ecliptic. This is because you “see” eclipses – and thus the ecliptic - suppressed among the stars by the latitudinal component of the lunar parallax (see KPM:1980, p. 61 ff. & 81-84). So no wonder in Hellenistic astronomy the obliquity was set to 23;51,20º, too large by 11'.

7: Overview and outlook

Parameter	***Almagest***	**Reference**	**”Error”**	**Source**
Year, tropical	365;14,48d	365;14,33d	- 25'/cent..	Eclipses 6 & 11
Year, anomaly	do.	365;15,35d	77'/cent.	do.
Radix, tropical	330;45º	328;16º	2;29º	Seasons c. -150
Radix, anomaly	265;15º	272;12º	- 6;57º	6 & 11 symmetric
Apogee	65;30º	66º c. -150	- 0;30º	330;45º – 265;15º
”Precession”	1º/årh	1;23º/årh.	- 23'/cent.	Drift of eclipses
Max. Equation	2;23º	2;00º	+23'	Eclipse pattern
Obliquity	23;51,20º	23;40º	+11'	Eclipse parallax

The scheme surveys the foregoing results regarding fundamental parameters in the *Almagest*. In particular you learn that apart from the tropical radix value all the basic parameters are founded on lunar eclipse data. Taking this into account more or less repairs the deviations labelled “errors” with reference to the solar motion.

Note also that the “errors” of the supposed tropical solar motion and of “precession” are coupled so as to produce perfectly sound sidereal longitudes, not only of Sun and Moon, but also of the planets, viz. using the synodic period relations of the planetary anomalies introduced in part 1 above. However, it was left to Medieval and Renaisance astronomers to exploit this. From 9th century Baghdad onwards you meet theories of a multicomponent motion of the eighth “starry” sphere. Thus a correspondence was established with the sidereal kernel of the ancient doctrine; but on the cosmological front you had to invent an additional sphere for each new component of the “motus octavae sphaerae” (KPM: 1974a). Eventually Copernicus in 16th century Frauenburg built a similar correspondence with ancient sidereal reality, but he turned traditional cosmology upside down and explained the matter by minutely tilting the Earth's axis and equator. So he bridged the gap between the Renaissance obliquity of the ecliptic and its “erroneous” ancient counterpart and at the same time he introduced unequal precession of the equinoxes against a supposed unshakeable background of the starry heaven (KPM: 1968 & 1974b). As we learn from G. J. Rheticus in *Narratio Prima* this very business played a key role for Copernicus' work on his new cosmology:

> *The Principal Reasons Why We Must Abandon the Hypotheses of the Ancient Astronomers*
>
> In the first place, the indisputable precession of the equinoxes, as you have heard, and the change of the obliquity of the ecliptic persuaded my teacher to assume that the motion of the earth could produce most of the appearences in the heavens, or at any rate save them satisfactorily (Rosen:1959, p. 136).

Here you face a practise of developing astronomical doctrines by way of building correspondence with earlier theories. I find similar traces of correspondence involved in the proces of transition from synodic “prototables” counting time by months to the Hellenistic *Almagest* tables with their day by day time measure. Since the empirical foundation of this originates in lunar eclipse reports, the most basic and reliable relation must be: 5458 months go with 5923 and 465 draconitic revolutions of Moon and Sun respectiely, or because you have two eclipse warnings per draconitic revolution: 465 eclipse warnings per 2729 months. So the most usable eclipse periods emerge as the continued fraction convergents:

6/1, 41/7, 47/8 = 235/40, 88/15, 135/23, 223/38 (Saros), 358/61, 2729/465

We have already found the fraction 2729/465 extremely accurate. Indeed, modern tables (KPM:1975c) yield 3087/526, or an average interval between eclipse warnings 10 seconds longer than the Babylonian result. Let it be too much to ask for agreement dead on. But remarkably the said convergents go with “round” half degree additions to hit already at the outset the final draconitic motions to the nearmost minute of arc, (0,5*360º + 4º)/6 = 30;40º/Ms for the Sun, and therefore 390;40º/Ms for the Moon. Moreover the following entries hit the goal to within about a second of arc:

(3,5*360º – 2,5º)/41 = 30;40,14.6º/Ms, (4*360º + 1,5º)/47 = 30;40,12.8º/Ms

(7,5*360º – 1º)/88 = 30;40,13.6º/Ms, (11,5*360º + 0,5º)135 = 30;40,13.3º/Ms

(19*360º -0,5º)/223 = 30;40,13.5º/Ms (30,5*360º + 0º)/358 = 30;40,13.4º/Ms

2961,5*360º/2729 = 30;40,14.1º/Ms

The Saros period of course stands out by its resonance with the period of lunar anomaly. But this has only limited influence on full moon and eclipse locations. So here we shall only mention the very short string of convergents, 13/14, 14/15, 251/269. Thus adding to 13 months multiples of 14 months you produce a series usable relations: 223/239 (Saros), 237/254, 251/269, where the first term is also an eclipse period, and the second establishes a connection to the 235 full moon positions since the Moon per month moves through 254 steps. So an anomalistic revolution of the Moon covers 237 steps.

The period of 235 months played a key role in building the above doctrine of astronomy based of lunar eclipse reports with the full moon serpent as the underlying conceptual vehicle. However, here the 235 months interval enters the story in fifths, namely via 47 months equal to 19/5 “synodic” years. This may, or may not, bear on the geared device from second century BC. Greece, known as the Antikythera Mechanism with its dial of 235 months divided into five 47-month turns of a spiral, and supplemented by a Callipic dial of four 235-month cycles. Anyway it was by way of the Antikythera machine, that I became aware of the overwhelming beauty of the eclipse patterns connected with the period of 47 months; see appendix A,

Column E of the system A lunar theory forms a crowning achievement of Babylonian astronomy. Combining the very accurate draconitic rate of motion with the canonical “year” of 12;22,08 = 2783/225 months produces a fully-fledged eclipse theory. Connected to the skew distribution in the zodiac of full moon and eclipse locations the year should be taken as anomalistic. So it is rather sound deviating -5,3'/century only. Counted as sidereal the year deviates by -24,4'/century. So this may explain why the Greeks preferred the sidereal year of 1979/160 months (or the 1º/century precession together with the synodic year 235/19 months). Anyway the Column E data may be interpreted in terms of a Babylonian Full Moon Serpent ; see Appendix B and (KPM:1980, p. 92-94).

8: References

(Freeth, T. et al:2006) “Decoding the ancient Greek astronomical calculator known as the Antikythera Mechanism”. *Nature* **444**, 587-591.

(H. H. Goldstine:1973) “New and Full Moons 1001 B.C. To A.D. 1651”. Philadelphia.

(A. Jones & D. Duke:2005) “Ptolemy's Planetary Mean Motions Revisited”. Centaurus, vol. 47, 226-35.

(KPM:1968) "The 1717 Egyptian Years and the Copernican Theory of Precession". Centaurus XIII, 120-138.

(KPM:1974a) "Thabit ibn Qurra between Ptolemy and Copernicus: An Analysis of Thabit,s Solar Theory". Archive for History of Exact Sciences XII, 199-216.

(KPM:1974b) "Success and Failure in Copernicus' Planetary Theories". Archives Internationales d'Histoire des Sciences XXIV, 73-111 & 243-318.

(KPM:1975a) "From Copernicus to Tycho Brahe. Or from blind Trusting to frank Rejection of ancient records of observation". Avant, avec, aprés Copernic (Paris) 187-190.

(KPM:1975b) "Hunting the Immobile Centre of Copernicus' Planetary System". Studia Copernicana XIV = Colloquia Copernicana IV, 93-100.

(KPM:1975c) "Elements of PLanetary, Lunar, and Solar Orbits, 1900 B.C. to A.D. 1900, Tabulated for Historical Use". Centaurus XIX, 157-181.

(KPM:1976) "The Bright Stars of the Zodiac, A Catalogue for Historical Use". With an appendix by Leif Kahl Kristensen. Centaurus XX, 129-158.

(KPM:1980) "The Full Moon Serpent. A Foundation Stone of Ancient Astronomy?". Centaurus XXIV, 51-96.

(KPM:1983a) "Basic Units in Chronology and Chronometry". i Gregorian Reform of the Calendar (ed. G.V. Coyne et al.) Citta del Vaticano, 3-14.

(KPM:1983b) "Ancient Ephemeris Time in Babylonian Astronomy", Journal for the History of Astronomy, XIV, 47-60.

(KPM:1985) "Synodic Period Relations in Babylonian and Hellenistic Astronomy". Vistas in Astronomy, XXVIII, 119-121.

(KPM:1987) "In Chase of an Origin for the Mean Planetary Motions in Ptolemy's *Almagest*". i From Ancient Omens to Statistical Mechanics (eds. J.L. Berggren and B.R. Goldstein) Copenhagen, 43-54.

(R. R. Newton:1976) “Ancient Planetary Observations and the Validity of Ephemeris Time”. The John Hopkins University Press, Baltimore and London. - Reviewed by KPM in ???

(R. R. Newton:1977) “The Crime of Claudius Ptolemy”. The John Hopkins University Press, Baltimore and London. - Reviewed by KPM in JHA, xi (1980), 133-135.

(O. Neugebauer:1963) New Edition of “Des Claudius Ptolemäus Handbuch der Astronomie, übersetzt und mit erklärenden Anmerkungen versehen von Karl Manitius, I-II” Leipzig 1912-13.

(Oppolzer, Th. R. v:1887) *Canon der Finsternisse*, Vienna.

(O. Pedersen:1974) “A Survey of the Almagest”, Odense University Press.

Ptolemy, “Almagest” see (O. Neugebauer:1963) and (G. J. Toomer:1984)

(E. Rosen: 1959) Three Copernican Treatises: The *Commentariolus* of Copernicus, The *Letter against Werner*, The *Narratio Prima* of Rheticus: 2nd Edition ... Dover Publications, New York.

(G. J. Toomer:1984) “Ptolemy's ALMAGEST, Translated and Annotated” Duckworth, London.

(A. J. Sachs, H. Hunger: 1988, vol.1 & 1989, vol.2) “Astronomical Diaries and Related Texts from Babylonia” Verlag der Österreichischen Akademie der Wissenschaften.

9: Appendix A: The 47-months cycle and the Antikythera mechanism

The period of 235 months played a key role in building the above doctrine of astronomy based of lunar eclipse reports with the full moon serpent as the underlying conceptual vehicle (KPM:1980 &1983). However, here the 235 months interval enters the story in fifths, namely as 47 months equal to 19/5 “synodic” years. Our Figure displays the “full moon serpent” from -468, January 24 (GN 6580) through -450, December 25 (GN 6814). Numbering from the beginning of Aries full moon positions by n = 1,2,..., 235 and possible eclipse locations by N = 0, 1,..., 34, we may single out any eclipse warning by the triple number set (GN, n, N)

with $n(GN) = (GN + 4)*19 \mod. 235$

and $N(GN)$ can be determined from equation (4)

Arranging now the 47 month cycles in five strips creates a most beautiful eclipse pattern. The 35 knots of the Full Moon Serpent split into 7 pentagonal sets, A, B ... G, of eclipse warning positions. Each set includes five members with 47 n-steps between any two neighbours:

A: N = 0, 7, 14, 21, 28 B: N = 1, 8, 15, 22, 29

C: N = 2, 9, 16, 23, 30 D: N = 3, 10, 17, 24, 31

E: N = 4, 11, 18, 25, 32 F: N = 5, 12, 19, 26, 33

G: N = 6, 13, 20, 27, 34

	A	D	G	C	(F	(B	X	B)	E)
6580	6581 095/14	6587 209/31	6593 088/13	6599 202/30	6605 081/12	6611 195/29		6616	6622
6627	6628 048/07	6634 162/24	6640 041/06	6646 155/23	6652 034/05	6658 148/22		6663	6669
6674	6675 001/00	6681 115/17	6687 229/34	6693 108/16	6699 222/33	6705		6710	6716 075/11
6721	6722 189/28	6728 068/10	6734 182/27	6740 061/09	6746 175/26	6752		6757 149/22	6763 028/04
6768	6769 142/21	6775 021/03	6781 135/20	6787 014/02	6793	6799		6804 102/15	6810 216/32

Figure 1: Five 47-month strips giving dates by lunation number, GN = 6580, ..., 6814, longitudes by Meton step, n = 1, ..., 235, and eclipse warnings by knot number, N = 0, ..., 34 for the full moons of the period from 469 through 451 BC. The headings of the eclipse colums refer to seven pentagonal sets of eclipse locations in the zodiac.

The net change of full moon position is 19 n-steps per lunation. So after 47 synodic months the shift amounts to 47*19 = 4*235 - 47 steps. In other words the lapse of one fifth of a 235 months cycle causes the full moon to move “backwards” by one fifth of a revolution in the zodiac, i.e. it sticks to the same set of eclipse warnings. The result is shown in Figure 1 with the actual occurrence of eclipses marked by shading in grey. It appears that the columns go with the positional groups above in the following order: A, D, G, C, (F, (B) & E), where the brackets signify that the group F runs short of eclipses and the group E starts void of eclipses, whereas the eclipse warnings of group B is divided between to neighbouring sets of full moons one n-step apart.

Note in particular that neither of the full moons (6658, 148, 22) and (6757, 149, 22), marked in blue in the figure, is eclipsed. During the intervening 99 months the change of draconitic longitude is about 107,5*360 - 23;30 degrees. By reason of symmetry the two full moons must needs occur about 11;45° after and before the relevant node of the lunar orbit. This sets an upper eclipse limit for the distance between the full moon and the node of the lunar orbit.

Because the entire eclipse knot pattern drifts by one step per 65 years we can be sure that after 76-years, or 4 cycles all of the eclipsing knots have moved at least one step. Figure 2 illustrates how this happened by moving the five month interval and its twin columns through the groups of 47 month eclipse intervals. The extended eclipse columns in Figure 2 show how the eclipse triples coalesce into a series of 15 members at 47-month intervals, e.g. the F-column from GN2825 to GN3483 and again the C-column from GN2960 to GN3618. By and large each series has central eclipses in the middle and small ones at both ends. But the full moon positions and the eclipse warnings are not spread evenly along the zodiac as they are affected by the *annual* inequality of the solar (and lunar) motion. However, with due regard for the inequality of the solar motion you may predict close values of lunar eclipse magnitudes from the scheme (KPM:1980 & 1983).

	F	B	E	A	D	(G	X G)	C)
2820	2825 171/26	2831 050/08	2837 164/25	2843 043/07	2849 157/24	2855 036/06	2860	2866
2867	2872 124/19	2878 003/01	2884 117/18	2890 231/00	2896 110/17	2902 224/34	2907	2913
2914	2919 077/12	2925 191/29	2931 070/11	2937 184/28	2943 063/10	2949 177/27	2954	2960 151/23
2961	2966 030/05	2972 144/22	2978 023/04	2984 137/21	2990 016/03	2996	3001	3007 104/16
3008	3013 218/33	3019 097/15	3025 211/32	3031 090/14	3037 204/31	3043	3048 178/27	3054 057/09

	F	B	E	(A	(D	X D)	G	C
3055	3060 171/26	3066 050/08	3072 164/25	3078 043/07	3084 157/24	3089	3095 131/20	3101 010/02
3102	3107 124/19	3113 003/01	3119 117/18	3125 231/00	3131	3136	3142 084/13	3148 198/30
3149	3154 077/12	3160 191/29	3166 070/11	3172	3178	3183 158/24	3189 037/06	3195 151/23
3196	3201 030/05	3207 144/22	3213 023/04	3219	3225	3230 111/17	3236 225/34	3242 104/16
3243	3248 218/33	3254 097/15	3260 211/32	3266	3272	3277 064/10	3283 178/27	3289 057/09

	F	(B	(E X	E)	A	D	G	C
3290	3295 171/26	3301 050/08	3307 164/25	3312	3318 138/21	3324 017/03	3330 131/20	3336 010/02
3337	3342 124/19	3348 003/01	3354	3359	3365 091/14	3371 205/31	3377 084/13	3383 198/30
3384	3389 077/12	3395 191/29	3401	3406 165/25	3412 044/07	3418 158/24	3424 037/06	3430 151/23
3431	3436 030/05	3442	3448	3453 118/18	3459 232/00	3465 111/17	3471 225/34	3477 104/16
3478	3483 218/33	3489	3495	3500 071/11	3506 185/28	3512 064/10	3518 178/27	3524 057/09

	(F X	F)	B	E	A	D	G	(C
3525	3530 171/26	3535	3541 145/22	3547 024/04	3553 138/21	3559 017/03	3565 131/20	3571 010/02
3572	3577	3582	3588 098/15	3594 212/32	3600 091/14	3606 205/31	3612 084/13	3618 198/30
3619	3624	3629 172/26	3635 051/08	3641 165/25	3647 044/07	3653 158/24	3659 037/06	3665
3666	3671	3676 125/19	3682 004/01	3688 118/18	3694 232/00	3700 111/17	3706 225/34	3712
3713	3718	3723 078/12	3729 192/29	3735 071/11	3741 185/28	3747 064/10	3753 178/27	3759

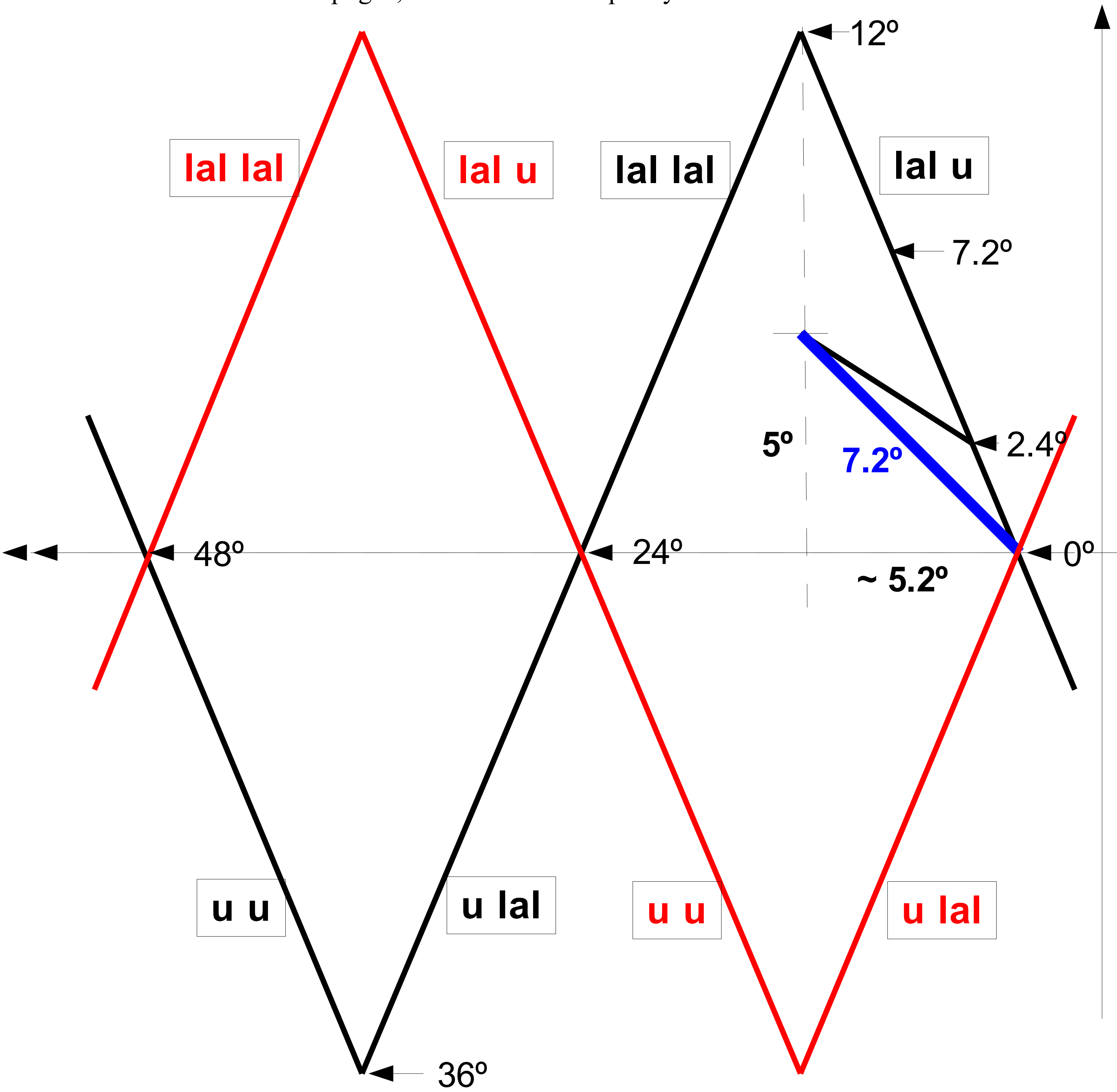
Figure 2: The lunar eclipses (Oppolzer 1887) of a Callipic period from 773 through 698 BC, arranged in four 235 months schemes like Figure 1.

10: Appendix B: Babylonian column E and the Full Moon Serpent

Behind the Babylonian column E you find the relations:

5458 Ms = 5923 Md = 465 Yd & 2783 Ms = 3008 M? = 225 Y?

It is commonplace to think of the year, 2783/225 = 12;22,08 Ms, as sidereal, and possibly ancient astronomers would agree. Nevertheless I shall here defend its "anomalistic" nature because of its role in building a nice lunar eclipse pattern. If so the month shall also be "anomalistic", but in the sense only that the "lunar" motion is referred to the "solar" apogee, i.e. the annual inequality of lunar motion is included.



Traditionally col. E is held to carry lunar latitudes and to produce eclipse warnings near the nodes of the lunar orbit. However, we may as well imagine the Babylonians ignorant of the concept of a lunar orbit, not to speak of its nodes. Instead we shall investigate eclipse warnings by way of the 35 quasistationary eclipsing “knots” spread along the Full Moon Serpent, cf. Equation (4). Clearly a Babylonian “serpent” should show zig-zags instead of sinusoidal “waves”. As seen in the Figure the distance from a “knot” (at 0º) reaches a maximum of 12º, whence it decreases via the next following “knot” to a minimum of -12º. Finally it goes up to the third “knot” completing a full zig-zag of wavelength 48º. After 17,5 zig-zags making a full revolution another (red) wave begins at 0º, but shifted by half a wavelength. By hindsight we know that this reflects a shift between the nodes of the lunar orbit.

Clearly the “distance” between a “knot” and the full moon can never reach 12º. But Col. E yields sound results by halving whatever goes beyond 2,4º. So you get the maximum distance of 7,2º as in the text and in perfect agreement with a maximum full moon latitude of 5º. Wave phases and “knot” distances are correlated as given below:

xº in wave	0º-12º	12º-24º	24º-36º	36º-48º
dist. in text	xº lal u	(24-x)º lal lal	(x-24)º u lal	(48-x)º u u

So far we may proceed as in equation (4) counting revolutions, waves, and fraction of the last wave passed by the full moon during any relevant period. But with reference to the zig-zag-serpent we may as well think of the full moon phase in any zig-zag, ph, as strongly dependent on the full moon longitude, given in col. B, but with a small correction dependent on time, i.e. the lunation number GN:

$$ph(B, GN) = \text{fraction of } \{(2761/96 - B - 3757*GN/2400)/360\}$$

The midle term, B/360, reflects automatically the inequality due to solar anomaly, and the last term (by hindsight) goes with the monthly drift of the lunar nodes.

Now multiplication by the zig-zag “wavelength” 48 creates the actual col. E values,

$$\mathbf{x = 2761/720 - 2B/15 - 3757*GN/18000 + q*48, \qquad 0 \leq x \leq 48} \qquad \mathbf{(6)}$$

which produces rather reliable lunar eclipse magnitudes. E.g. The following table deals with the eclipses from the leftmost column of figure 2 in appendix 1 above. i.e. the full series of 15 eclipses with 47 months between them, and the transition to another series at the neighbouring position after a slip of 99 months. For each month the table brings (1) solar longitude (or rather anomaly!) according to Babylonian system A theory, (2) full moon longitude by addition of 180º, (3) the zig-zag phase x of the above equation, and (4) the distance from the nearmost knot according to the said correlation rules.

To evaluate the eclipse magnitudes inherent in the Col. E distances we may calculate

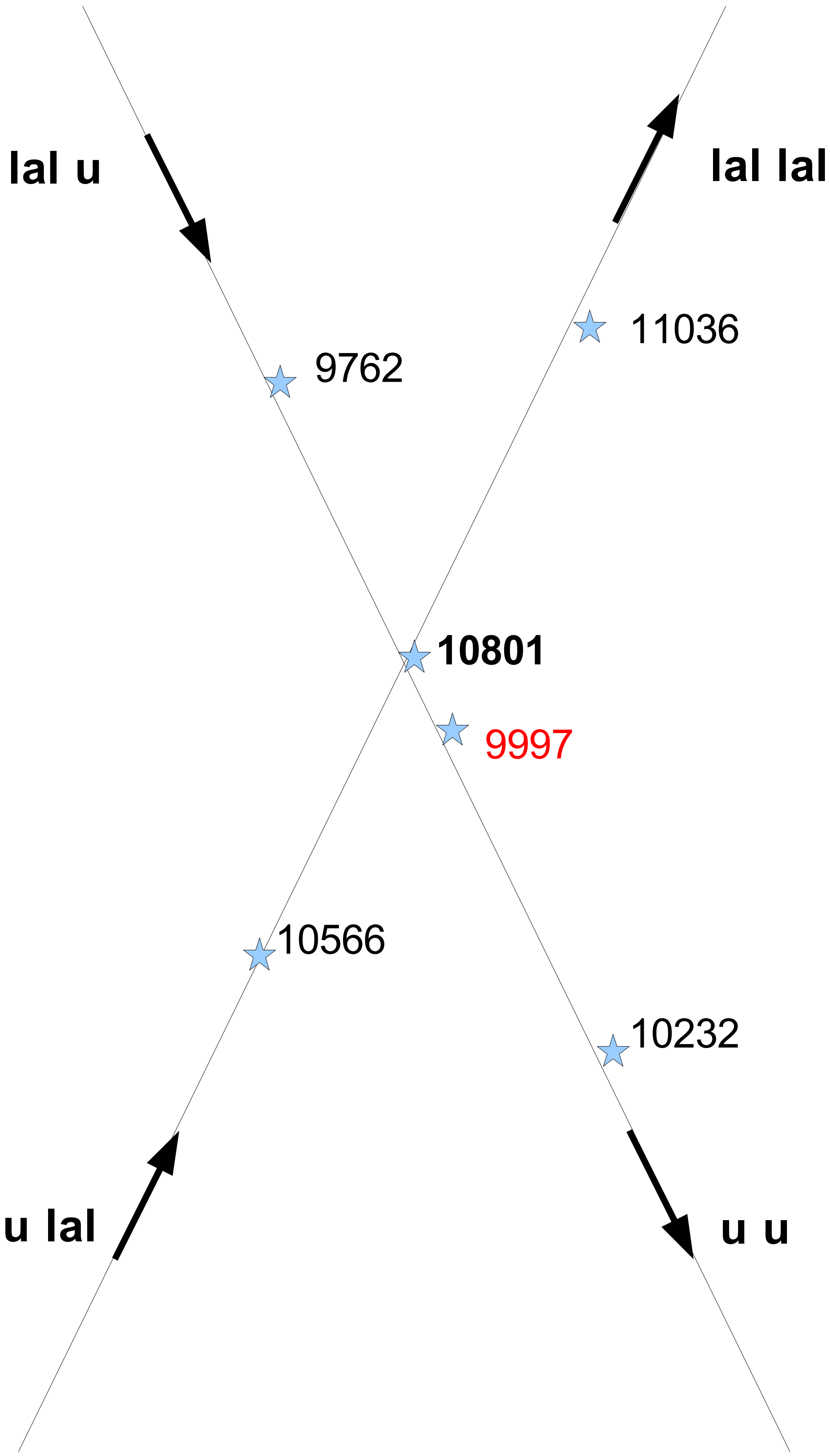
$$\textit{Magnitude} = 22 - \textit{distance}*60/4{,}2$$

in accordance with the eclipse limit close to the distance 1;32 = 22*4,2' as revealed at GN3530. The results compare favourably with Oppolzers table (Oppolzer: 1887), and they are certainly superior to the outcome of any linear relation. This works because the unequal motion of the **slow** sun locates correctly, **where** full moons may occur. The inequality of lunar motion enters the scene in timing the penomena. A full moon event happens, **when** the **swift** moon arrives at the right position.

GN	B(Sun)	B(Ecl)	x (zig-zag)	Distance (text)	Magn.	Oppolzer	
2825	95;07,30	275;07,30	1;30,40	1;30,40 lal u	0,4	-772JN	1,3
2872	25;30	205;30	0;59,04,12	0;59,04,12 lal u	7,9	-768AP	9
2919	313;08	133;08	0;49,24,24	0;49,24,24 lal u	10,2	-764JA	11,5
2966	238;52	58;52	0;54,56,36	0;54,36,36 lal u	9	-761NO	11,3
3013	164;36	344;36	1;00,28,48	1;00,28,48 lal u	7,6	-757SE	8,5
3060	94;52,30	274;52,30	0;29,41	0;29,41 lal u	14,9	-753JN	15,5
3107	25;15	205;15	47;58,05,12	0;01,54,18 u u	21,5	-749AP	21,4
3154	312;52	132;52	47;48,33,24	0;11,26,36 u u	19,3	-745JA	17,6
3201	238;36	58;36	47;54,05,36	0;05,54,24 u u	20,6	-742NO	18,4
3248	164;20	344;20	47;59,37,48	0;00,22,12 u u	21,9	-738SE	20,8
3295	94;37,30	274;37,30	47;28,42	0;31,18 u u	14,5	-734JN	13,5
3342	25	205	46;57,06,12	1;02,53,48 u u	7	-730AP	6
3389	312;36	132;36	46;47,42,24	1;12,17,36 u u	4,8	-726JA	3
3436	238;20	58;20	46;53,14,36	1;06,45,24 u u	6,1	-723NO	2,3
3483	164;04	344;04	46;58,46,48	1;01,13,12 u u	7,4	-719SE	6,4
3530	94;32,30	274;22,30	46;27,43	1;32,17 u u	0	-715JN,	- ecl.
+99 Ms							
3629	95;45	275;45	25;36,54,24	1;36,54,24 u lal	-1,1	- 707JN,	- ecl.
3676	26;07,30	206;07,30	25;05,18,36	1;05,18,36 u lal	6,5	-703AP	7,9

Now we shall investigate the events at a definite "knot" where we know that eclipses occur in triads which in their turn drifts by some "step" every 65 years, or 804 months: 804*225 = 65*2783 + 5. Let us begin with the "knot" 20 eclipse at GN 9997, which for reasons of symmetry appeared central - in the *Almagest* and in Oppolzer. Moreover it turned up again behind the Saturn tables of anomaly, cf. p. 5. However, the following table shows that the col. E theory goes with the assumption of a central eclipse at GN 10801 = 9997 + 804. This is illustrated in the figure on the next page where we also learn that the former "lal" and "u" refer to *positive* and *negative* latitude, and the latter "lal" or "u" mean on its way *up* or *down* respectively.

GN	B(Sun)	B(Ecl.)	x (zig-zag)	Distance (text)	Magn.	Remarks
9762	40;22,30	220;22,30	0;54,18,12	0;54,18,12 lal u	9,1	
9997	40;07,30	220,07,30	47;53,19,12	0;06,40,48 u u	20,4	cf. equ. (4)
10232	39;5230	219;52,30	46;52,20,12	1;07,39,48 u u	5,9	*Alm.* no. 33
10566	41	221	25;00,32,36	1;00,32,36 u lal	7,6	
10801	40;45	220;45	23;59,33,36	0;00,26,24 lal lal	21,9	Central
11036	40;30	220;30	22;58,34,36	1;01,25,24 lal lal	7,4	



Altogether Col. E makes up, indeed, a fullyfledged Baylonian Full Moon Serpent.

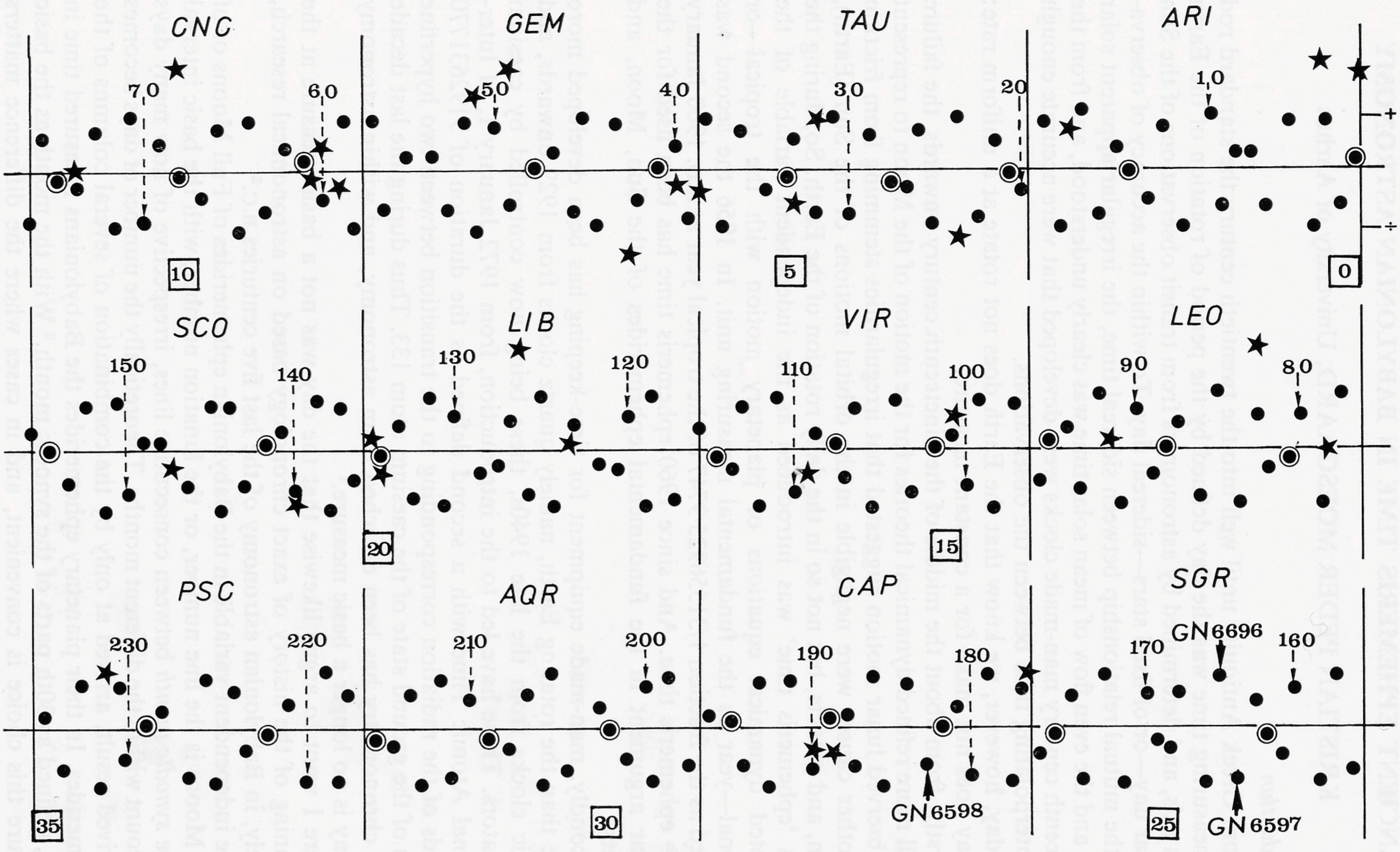


FIG. 1. Map of the zodiac for the epoch 450 B.C., with positions of the 235 Full Moons from 469 to 451 B.C. Lunar eclipses are marked by circles around the Full Moon dots.

RePoSS issues

#1 (2008-7) **Marco N. Pedersen**: "Wasan: Die japanische Mathematik der Tokugawa Ära (1600-1868)" (Masters thesis)

#2 (2004-8) **Simon Olling Rebsdorf**: "The Father, the Son, and the Stars: Bengt Strömgren and the History of Danish Twentieth Century Astronomy" (PhD dissertation)

#3 (2009-9) **Henrik Kragh Sørensen**: "For hele Norges skyld: Et causeri om Abel og Darwin" (Talk)

#4 (2009-11) **Helge Kragh**: "Conservation and controversy: Ludvig Colding and the imperishability of "forces"" (Talk)

#5 (2009-11) **Helge Kragh**: "Subatomic Determinism and Causal Models of Radioactive Decay, 1903–1923" (Book chapter)

#6 (2009-12) **Helge Kragh**: "Nogle idéhistoriske dimensioner i de fysiske videnskaber" (Book chapter)

#7 (2010-4) **Helge Kragh**: "The Road to the Anthropic Principle" (Article)

#8 (2010-5) **Kristian Peder Moesgaard**: "Mean Motions in the *Almagest* out of Eclipses" (Article)

#9 (2010-7) **Helge Kragh**: "The Early Reception of Bohr's Atomic Theory (1913-1915): A Preliminary Investigation" (Article)

#10 (2010-10) **Helge Kragh**: "Before Bohr: Theories of atomic structure 1850-1913" (Article)

#11 (2010-10) **Henrik Kragh Sørensen**: "The Mathematics of Niels Henrik Abel: Continuation and New Approaches in Mathematics During the 1820s" (PhD dissertation)

#12 (2009-2) **Laura Søvsø Thomasen**: "In Midstream: Literary Structures in Nineteenth-Century Scientific Writings" (Masters thesis)

#13 (2011-1) **Kristian Danielsen and Laura Søvsø Thomasen (eds.)**: "Fra laboratoriet til det store lærred" (Preprint)

#14 (2011-2) **Helge Kragh**: "*Quantenspringerei*: Schrödinger vs. Bohr" (Talk)

#15 (2011-7) **Helge Kragh**: "Conventions and the Order of Nature: Some Historical Perspectives" (Talk)

#16 (2011-8) **Kristian Peder Moesgaard**: "Lunar Eclipse Astronomy" (Article)

#17 (2011-12) **Helge Kragh**: "The Periodic Table in a National Context: Denmark, 1880-1923" (Book chapter)

#18 (2012-1) **Henrik Kragh Sørensen**: "Making philosophy of science relevant for science students" (Talk)

RePoSS (Research Publications on Science Studies) is a series of electronic publications originating in research done at the Centre for Science Studies, University of Aarhus.

The publications are available from the Centre homepage (www.css.au.dk/reposs).

ISSN 1903-413X

Centre for Science Studies
University of Aarhus
Ny Munkegade 120, building 1520
DK-8000 Aarhus C
DENMARK
www.css.au.dk

Faculty of Science
Department of Science Studies
AARHUS UNIVERSITET